MR PAGE

PLANNING AND INSTALLATION DEPARTMENT
RESEARCH DEPARTMENT

"T"-TYPE COAXIAL NOTCH FILTERS FOR USE AT VHF

Report No. E-070

(1960/7)

THE BRITISH BROADCASTING CORPORATION
ENGINEERING DIVISION

PLANNING AND INSTALLATION DEPARTMENT

RESEARCH DEPARTMENT

"T"-TYPE COAXIAL NOTCH FILTERS FOR USE AT VHF

Report No. E-070

(1960/7)

(E.W. Hayes)

W Proctor Wilson

(W. Proctor Wilson)

W. Wharton, A.M.I.E.E.
R.E. Davies, B.A.

This Report is the property of the British Broadcasting Corporation and may not be reproduced in any form without the written permission of the Corporation.

Report No. E-070

"T"-TYPE COAXIAL NOTCH FILTERS FOR USE AT VHF

Section	Title	Page
	SUMMARY	1
1	INTRODUCTION	1
2	GENERAL THEORY OF CLOSE-SPACED NOTCH FILTERS	3
3	SOLUTION OF THE DESIGN EQUATIONS OF THE "T"-TYPE NOTCH FILTER	7
	3.1. Pass and Stop Conditions	7
	3.2. The Additional Condition	7
	3.3. Method of Solution	9
	3.4. Theory of Method of Solution	10
	3.5. Preparation of Graphs	12
4	USE OF DESIGN CURVES	12
5	CONCLUSIONS	18
6	REFERENCE	18

March 1960

Report No. E-070

(1960/7)

"T"-TYPE COAXIAL NOTCH FILTERS FOR USE AT VHF

SUMMARY

This report describes the theory of the "T"-type coaxial notch filter and presents design curves enabling the dimensions of practical filters to be calculated rapidly. The type of filter considered was first described by Sosin[1] and provides a two-terminal impedance with a pole and zero close together in frequency. If the two-terminal impedance is connected in shunt with a transmission line it forms a filter which will pass one frequency and stop another adjacent frequency. The underlying theory of filters of this type has been investigated by P.I.D. and filters for many practical applications have been designed. The equations governing the design, however, cannot be solved in terms of tabulated functions and it has been necessary to solve individual cases graphically. The equations have now been solved by Research Department for a useful range of values by an iterative method and design curves plotted.

1. INTRODUCTION

Many practical applications arise in v.h.f. work in which a two-terminal admittance with a closely spaced pole and zero is required. Such networks are used as simple filters to stop and pass desired frequencies when connected across a transmission line and also form a part of constant-impedance networks used to combine the outputs of two or more transmitters into a common aerial. For convenience the two-terminal networks will be referred to as "notch" filters, the frequency of the pole as the "stop" frequency and the frequency of the zero as the "pass" frequency.

At v.h.f. it is desirable to construct notch filters from coaxial elements and there are several configurations which will achieve the desired admittance characteristic. The most compact and flexible configuration of elements is that described by Sosin[1] which consists of a "T" of three coaxial elements of similar characteristic impedance as shown in Fig. 1.

In general it is convenient to construct the filter from coaxial elements of the same characteristic impedance as the transmission line across which it is connected, as illustrated in Fig. 1, and in this report it is assumed that the design is carried out in this way.

In the arrangement of Fig. 1 it is convenient first to consider the performance when the element θ_3 is of zero length. In this case, in the absence of losses, there will be an admittance zero when $\theta_1 + \theta_2 = \pi/2$ and an admittance pole when $\theta_1 = \pi/2$. It should be further noted that, for close frequency spacing of the pole and zero, the rate of change of susceptance with frequency at the zero will be large

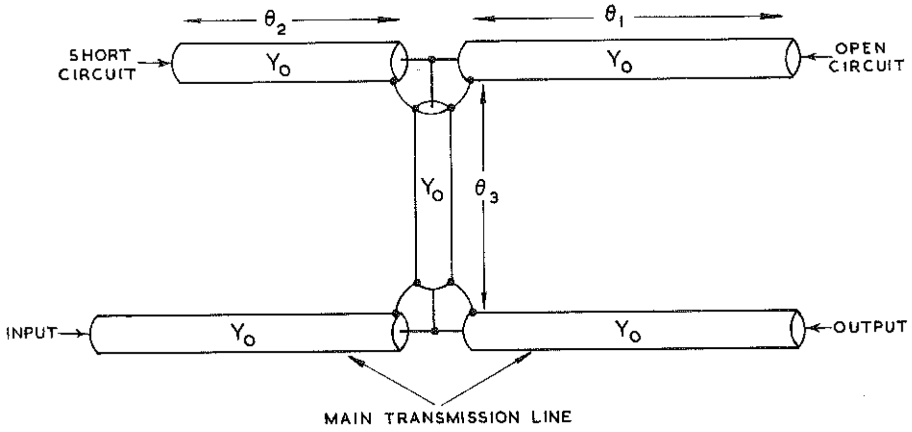

Fig. 1 - General arrangement of "T"-type coaxial notch filter

and the rate of change of reactance with frequency at the pole (impedance zero) will be small. This means that for signals transmitted along the main transmission line (see Fig. 1) the bandwidth in the pass condition will be narrow and the resistive loss high and the bandwidth in the stop condition broad and the resistive stop loss high.

If a length of feeder θ_3 (between 0 and $\pi/2$ radians in electrical length) is now added to the filter and the elements θ_1 and θ_2 are retuned to give a pole and zero at the same frequencies as before, the rates of change of susceptance and reactance at the admittance zero and pole respectively are found to be modified. The susceptance slope at the zero is reduced and the reactance slope at the pole is increased, and the resistive pass and stop losses both reduced. By the addition of suitable values of θ_3, therefore, it is possible, without losing the compact nature of the arrangement, to obtain a range of filters with the same desired pass and stop frequencies but with differing pass and stop bandwidths and pass and stop losses.

Unfortunately, in spite of its compactness and electrical simplicity, the "T"-type notch filter is difficult to design since the equations defining its performance are transcendental and cannot be solved explicitly. The filters have, in fact, been used by P.I.D. for some years, the design being carried out graphically or, in certain cases, by means of suitable approximations. It was felt, however, that a set of curves which would enable the design parameters for any particular case to be readily determined was most desirable and would result in a considerable saving of time. Discussions between P.I.D. and Research Department were therefore held and the problem was arranged by P.I.D. in a form suitable for mathematical solution by Research Department.

This report is divided into three sections, Section 2 contributed by P.I.D. describes the problem to be solved and the method by which the design parameters are determined, Section 3 contributed by Research Department describes the mathematical solution of the problem, and the production of the design curves. Section 4 describes, with examples, the use of the curves.

2. GENERAL THEORY OF CLOSE-SPACED NOTCH FILTERS

It has been pointed out, in Section 1 above, that a range of filter designs exists for specific values of pass and stop frequencies, and that any filter in this range can be obtained by a suitable choice of θ_3 if the "T"-type is used. In a practical filter, losses will occur, and the design chosen will be that giving a suitable compromise between:

(a) The diameter of the coaxial elements used.
(b) The power lost at the pass frequency.
(c) The attenuation obtained at the stop frequency.
(d) The required bandwidths at the pass and stop frequencies.

It is thus necessary to determine the following parameters before considering the solution of the equations governing the performance of any particular filter configuration:

(i) The pass frequency.
(ii) The stop frequency.
(iii) The relationship between the susceptance and reactance slopes at the pass and stop frequencies respectively i.e. the relationship between pass and stop bandwidths and losses.

If a filter is constructed of any number of elements which are all coaxial feeders of the same type, then irrespective of the configuration used, the admittance at the input terminals, as a function of frequency, may be taken as

$$Y = j\,Y_0\,F\left(\frac{2\pi f}{c} - j\,\alpha\right) \qquad (1)$$

where α is the loss of the cable used for the filter in nepers/unit length,
F is an arbitrary function depending upon the filter configuration,
Y_0 is the characteristic admittance of the coaxial feeder used for the elements,
f is the frequency and
c is the velocity of propagation of signals along the elements.

Equation (1) is valid for small cable losses due to series resistance, which condition is of interest in practice since cables involving appreciable shunt losses would be unsuitable for use in filters discriminating between closely spaced pass and stop frequencies.

If a "T"-type coaxial notch filter is designed to give a pole and zero of admittance spaced close together in frequency then any other poles and zeros occurring in the admittance/frequency characteristic will be distant in frequency from the required pole and zero by amounts large compared with the spacing between the required pole and zero. Equation (1) may, therefore, be rewritten in terms of the required pole and zero as follows

$$Y = \pm j\, Y_o\, N^{\frac{1}{2}}\, \frac{\frac{2\pi}{c}(f-f_p) - j\alpha}{\frac{2\pi}{c}(f-f_s) - j\alpha} \tag{2}$$

where f_s is the frequency of the pole (stop frequency), f_p is the frequency of the zero (pass frequency) and N is a constant governing the relationship between the bandwidths and losses at the stop and pass frequencies. The positive sign must be taken for $f_p > f_s$ and the negative in the contrary case. Hence, since α is small, the admittances at the pole and zero frequencies will be real and writing these as resistances we get

$$\frac{1}{Y}\text{ (stop frequency) } = r = \frac{\alpha c}{2\pi|f_p - f_s| Y_o N^{\frac{1}{2}}} \tag{3a}$$

$$\frac{1}{Y}\text{ (pass frequency) } = R = \frac{2\pi|f_p - f_s|}{\alpha c Y_o N^{\frac{1}{2}}} \tag{3b}$$

From Equations (3a) and (3b) two results can be obtained

$$\frac{1}{rR} = N Y_o^2 \tag{4a}$$

$$\frac{r}{R} = \frac{\alpha^2 c^2}{4\pi^2 (f_s - f_p)^2} \tag{4b}$$

If we write the ratio of the pole and zero frequencies as

$$k = \frac{f_p}{f_s}$$

then Equations (4) can be written

$$N = \frac{1}{Y_o^2\, rR} \tag{5a}$$

$$\frac{r}{R} = \frac{\alpha^2 \lambda^2}{4\pi^2 (1-k)^2} \tag{5b}$$

where λ is the wavelength at the stop frequency f_s, so that $f_s \lambda = c$.

In practice manufacturers quote the performance figures for the loss of cables in dB/100 ft. If, therefore, D is the loss in dB/100 ft and λ is measured in feet then Equations (5) become

$$N = \frac{1}{Y_o^2 rR} \qquad (6a)$$

$$D = \frac{5460(1-k)}{\lambda}\left(\frac{r}{R}\right)^{\frac{1}{2}} \qquad (6b)$$

Fig. 2 shows in schematic form a notch filter connected across a transmission line of characteristic impedance $Z_o = 1/Y_o$ and from this equivalent circuit the losses at the pass and stop frequencies will be

$$\text{Pass loss} = 20 \log_{10}\left(\frac{2RY_o}{1+2RY_o}\right) \text{ dB} \qquad (7a)$$

$$\text{Stop loss} = 20 \log_{10}\left(\frac{2rY_o}{1+2rY_o}\right) \text{ dB} \qquad (7b)$$

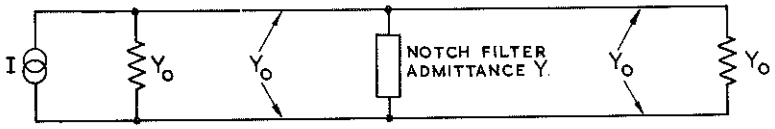

Fig. 2 - Equivalent circuit for calculating notch filter insertion losses

Equations (6) and (7) give a basis for determining the fundamental design of a notch filter whatever its configuration. Assuming that the required pass and stop losses are given and the required pass and stop frequencies are known the following calculations can be made:

(a) From Equations (7) (which are plotted in Fig. 3) the required values of rY_o and RY_o can be determined.

(b) From Equations (6b) and (6a) respectively the values of D, the cable attenuation and N may be determined and these values control the design of the actual filter to be used.

In designing the filter the correct size of cable can be determined from D, the required attenuation, but it is necessary to relate N to the filter configuration used. Since α is small, then from Taylor's theorem, the input admittance and impedance of the filter in Equation (1) may be written

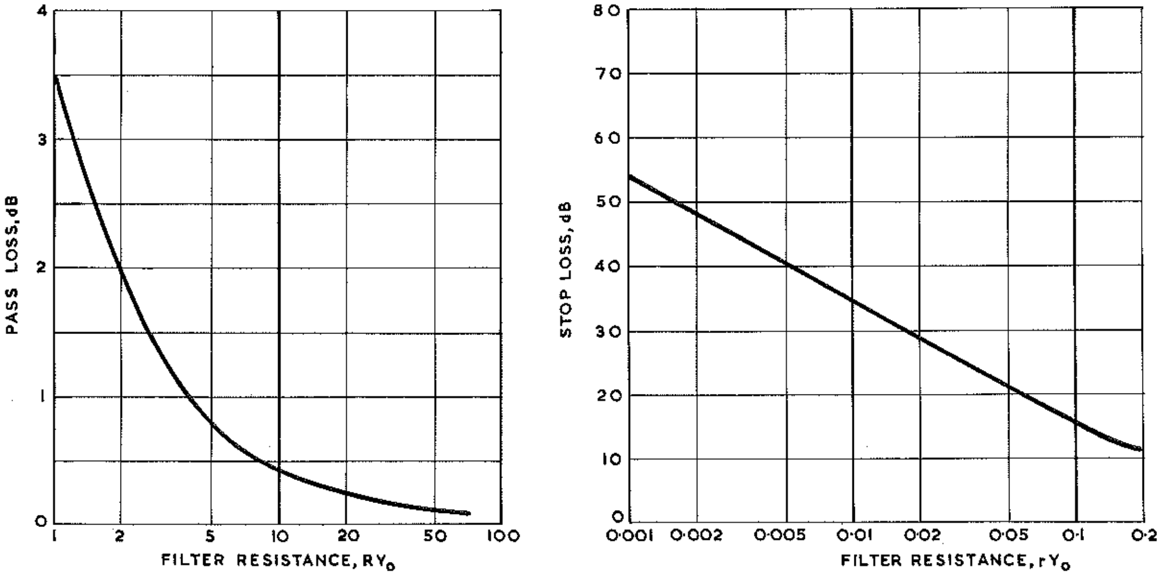

Fig. 3 - Relationship between filter resistance and pass and stop losses

$$\text{Admittance} = jY_0 F\left(\frac{2\pi f}{c}\right) + \frac{\alpha c}{2\pi}\frac{\partial}{\partial f}\left[Y_0 F\left(\frac{2\pi f}{c}\right)\right] \tag{8a}$$

$$\text{Impedance} = j\frac{1}{Y_0} F_1\left(\frac{2\pi f}{c}\right) + \frac{\alpha c}{2\pi}\frac{\partial}{\partial f}\left[\frac{1}{Y_0} F_1\left(\frac{2\pi f}{c}\right)\right] \tag{8b}$$

where $F_1\left(\frac{2\pi f}{c}\right) = -1/\{F(2\pi f/c)\}$

Thus from Equations (8a) and (8b) the values of resistance at the pole and zero are:

$$\left.\begin{aligned} R &= \frac{2\pi}{\alpha c \dfrac{\partial}{\partial f}[\text{susceptance}]_{f = f_p}} \\ r &= \frac{\alpha c \dfrac{\partial}{\partial f}[\text{reactance}]_{f = f_s}}{2\pi} \end{aligned}\right\} \tag{9}$$

Hence from Equation (6a):

$$N = \frac{1}{Y_0^2 Rr} = \frac{1}{Y_0^2}\frac{\dfrac{\partial}{\partial f}[\text{susceptance}]_{f = f_p}}{\dfrac{\partial}{\partial f}[\text{reactance}]_{f = f_s}} \tag{10}$$

For the design of a filter of any particular configuration, therefore, a knowledge of the required loss in dB/100 ft determines the type of cable to be used and a knowledge of the parameter N determines the particular filter required out of the range of filters, all of which pass and stop the required frequencies.

3. SOLUTION OF THE DESIGN EQUATIONS OF THE "T"-TYPE NOTCH FILTER

3.1. Pass and Stop Conditions

As in the preceding section let f_s be the stop frequency and f any other frequency, the ratio f/f_s being denoted by x. Then, if θ_1, θ_2 and θ_3 denote the electrical lengths of the arms of the filter as shown in Fig. 1 at frequency f_s the lengths at frequency f are $x\theta_1$, $x\theta_2$ and $x\theta_3$. Let k be the value of x at f_p the pass frequency. Then $f_p/f_s = k$, k being nearly unity. We shall consider mainly the case $k < 1$.

Application of the transmission line equations shows that, neglecting cable losses, the input admittance of the filter, denoted by jY,* is determined from the relations:

$$\left.\begin{aligned} Y &= Y_o \tan(\phi_x + x\theta_3) \\ \tan\phi_x &= \tan x\theta_1 - \cot x\theta_2 \end{aligned}\right\} \qquad (11)$$

where Y_o is the characteristic admittance of the cable. At the pass frequency

$$Y = 0, \qquad x = k$$

$$\therefore \ \tan(\phi_k + k\theta_3) = 0$$

$$\therefore \ \tan\phi_k = -\tan k\theta_3$$

Hence

$$\tan k\theta_1 - \cot k\theta_2 = -\tan k\theta_3 \qquad (12)$$

Similarly, at the stop frequency $x = 1$ and Y is infinite, or $\phi_1 + \theta_3 = \pi/2$. Hence

$$\tan\theta_1 - \cot\theta_2 = \cot\theta_3 \qquad (13)$$

In any particular design k is given and θ_1, θ_2 and θ_3 have to be determined. The Equations (12) and (13) will have a single infinity of solutions between 0 and $\pi/2$ (which is the range of interest since short cable lengths are desirable) and thus a finite number of solutions will be expected if one further condition is imposed.

3.2. The Additional Condition

As discussed in Section 2, from practical considerations it is desirable to control the losses and/or the bandwidths at the pass and stop frequencies. It is shown in Section 2 of this report that the parameter N is simply related to these losses and has the further advantage of being independent of the cable loss. A third condition is therefore imposed by specifying the value of N, from Equation (10) Section 2, which may also be written

---

*Because we work with the admittance jY rather than the susceptance Y, we get the extra minus in Equations 14.

$$N = -\frac{1}{Y_o^2}\,\frac{\left[\dfrac{\partial Y}{\partial f}\right]_{x=k}}{\left[\dfrac{\partial}{\partial f}\left(\dfrac{1}{Y}\right)\right]_{x=1}}$$

Y being defined by (11). It follows that

$$N = -\frac{1}{Y_o^2}\,\frac{\left[\dfrac{dY}{dx}\right]_{x=k}}{\left[\dfrac{d}{dx}\left(\dfrac{1}{Y}\right)\right]_{x=1}} \qquad (14)$$

θ_1, θ_2 and θ_3 being considered constant and Y a function of the variable x only.

Since

$$Y = Y_o \tan(\phi_x + x\theta_3)$$

$$\frac{dY}{dx} = Y_o \sec^2(\phi_x + x\theta_3)\,\frac{d}{dx}\,(\phi_x + x\theta_3)$$

$$= Y_o\left[1 + \frac{Y^2}{Y_o^2}\right]\frac{d}{dx}\,(\phi_x + x\theta_3) \qquad (15)$$

and

$$\frac{d}{dx}\left(\frac{1}{Y}\right) = -\frac{1}{Y_o}\,\mathrm{cosec}^2(\phi_x + x\theta_3)\,\frac{d}{dx}\,(\phi_x + x\theta_3)$$

$$= -\frac{1}{Y_o}\left[1 + \frac{Y_o^2}{Y^2}\right]\frac{d}{dx}\,(\phi_x + x\theta_3) \qquad (16)$$

Inserting expressions (15) and (16) into (14) we find

$$N = \frac{\left[\dfrac{d}{dx}\,(\phi_x + x\theta_3)\right]_{x=k}}{\left[\dfrac{d}{dx}\,(\phi_x + x\theta_3)\right]_{x=1}} \qquad (17)$$

since

$$Y = 0 \text{ at } x = k$$

and

$$\frac{1}{Y} = 0 \text{ at } x = 1.$$

Now

$$\frac{d}{dx}\,(\phi_x + x\theta_3) = \frac{d\phi_x}{dx} + \theta_3 \qquad (18)$$

Moreover, since $\tan\phi_x = \tan x\theta_1 - \cot x\theta_2$

$$\sec^2\phi_x \frac{d\phi_x}{dx} = \theta_1 \sec^2 x\theta_1 + \theta_2 \operatorname{cosec}^2 x\theta_2$$

$$\therefore \frac{d\phi_x}{dx} = \frac{\theta_1 \sec^2 x\theta_1 + \theta_2 \operatorname{cosec}^2 x\theta_2}{1 + \tan^2\phi_x}$$

$$= \frac{\theta_1 \sec^2 x\theta_1 + \theta_2 \operatorname{cosec}^2 x\theta_2}{1 + (\tan x\theta_1 - \cot x\theta_2)^2}$$

Putting $x = k$ and using (12) we find

$$\left(\frac{d\phi_x}{dx}\right)_{x=k} = \frac{\theta_1 \sec^2 k\theta_1 + \theta_2 \operatorname{cosec}^2 k\theta_2}{1 + \tan^2 k\theta_3} = \frac{\theta_1 \sec^2 k\theta_1 + \theta_2 \operatorname{cosec}^2 k\theta_2}{\sec^2 k\theta_3}$$

Similarly at $x = 1$

$$\left(\frac{d\phi_x}{dx}\right)_{x=1} = \frac{\theta_1 \sec^2\theta_1 + \theta_2 \operatorname{cosec}^2\theta_2}{1 + \cot^2\theta_3} = \frac{\theta_1 \sec^2\theta_1 + \theta_2 \operatorname{cosec}^2\theta_2}{\operatorname{cosec}^2\theta_3}$$

Hence from (17) and (18)

$$N = \frac{\operatorname{cosec}^2\theta_3}{\sec^2 k\theta_3}\left(\frac{\theta_1 \sec^2 k\theta_1 + \theta_2 \operatorname{cosec}^2 k\theta_2 + \theta_3 \sec^2 k\theta_3}{\theta_1 \sec^2\theta_1 + \theta_2 \operatorname{cosec}^2\theta_2 + \theta_3 \operatorname{cosec}^2\theta_3}\right) \tag{19}$$

3.3. Method of Solution

Since no explicit solution of Equations (12), (13) and (19) in terms of simple functions was known which gives θ_1, θ_2 and θ_3 in terms of k and N, iterative approximation methods were used to obtain a large number of corresponding sets of values of k, θ_1, θ_2 and θ_3 and from each set the value of N was found from Equation (19). Graphs were then produced by means of which the process of obtaining θ_1, θ_2 and θ_3 knowing k and N could be carried out immediately.

Since there is a double infinity of corresponding sets of k, θ_1, θ_2 and θ_3 any two of them may be chosen arbitrarily. The iterations consisted of finding closer and closer approximations to the other two until there is certainty of being within the pre-assigned tolerance. Two methods of iteration were used according to the range of values of θ_3 under consideration. These will be described in detail.

METHOD (1)

k and θ_1 are first chosen and approximations to θ_2 and θ_3 found by the following cycle of operations:

(1) Start with any value of θ_2, say θ_2'.

(2) Determine θ_3' from $\cot\theta_3' = \tan\theta_1 - \cot\theta_2' \ldots$ (A).

(3) Determine θ_2'', the next approximation to θ_2, from
$\cot k\theta_2'' = \tan k\theta_1 + \tan\theta_3' \ldots$ (B).

(4) Return to Equation (A) to determine a new approximation to θ_3, say θ_3'', using θ_2'' instead of θ_2'.

(5) Continue the cycle until successive approximations to θ_2 and θ_3 differ by less than 0·1°.

METHOD (2)

θ_1 and θ_2 are chosen initially, θ_3 being found directly from Equation (12). The iterative procedure to find k is as follows:

(1) Select k_1, the first approximation to k.

(2) Determine k_2 from

$$\tan k_{n+1}\theta_1 = \cot k_n\theta_2 - \tan k_n\theta_3 \quad \ldots \text{ (C)}$$

with $n = 1$.

(3) Determine new values k_{n+1}, from Equation (C) with $n = 2, 3, 4, 5 \ldots$ until the difference between successive approximations is less than 0·0002.

3.4. Theory of Method of Solution

Iterative methods of this kind are of very general application to the approximate solution of equations but the conditions of convergence (and preferably rapid convergence) require investigation.

For a function of one variable we have by Taylor's theorem

$$f(a+h) \approx f(a) + hf'(a)$$

provided $|h|$ is sufficiently small. Thus if $x = a$ is a solution of $f(x) = 0$ and a_n is our nth approximation to a

$$f(a_n) \approx (a_n - a)f'(a_n) \text{ or } a = a_n - \frac{f(a_n)}{f'(a_n)}$$

Hence we take

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

This method is usually known as Newton's method of successive approximation to the root of an equation.

The rapidity of convergence is assured if we start with a reasonably near approximation and if $f'(a_n)$ is fairly large.

If we have two equations involving two variables say $f(x, y) = 0$ and $g(x, y) = 0$ we may write

$$x_n = x + \epsilon_n$$
$$y_n = y + \delta_n$$

where the values x and y satisfy the equations. Then by Taylor's theorem

$$f(x_n, y_n) \approx f(x, y) + \epsilon_n f_x + \delta_n f_y$$

$$\left(f_x \text{ denotes } \frac{\partial f}{\partial x} \text{ and } f_y , \frac{\partial f}{\partial y} \right)$$

$$= \epsilon_n f_x + \delta_n f_y \text{ since } f(x, y) = 0$$

Similarly $$g(x_n, y_{n+1}) = \epsilon_n g_x + \delta_{n+1} g_y$$

Thus, if x_n is determined from y_n and $f(x_n, y_n) = 0$ and y_{n+1} from x_n and $g(x_n, y_n) = 0$ the iteration will converge provided that

$$\left| \frac{\delta_{n+1}}{\delta_n} \right| < 1 ,$$

the convergence being more rapid the smaller the ratio

$$\left| \frac{\delta_{n+1}}{\delta_n} \right| = \left| \frac{f_y g_x}{f_x g_y} \right|$$

Moreover, if we reverse the order of determination of the variables i.e. determine y_n from $f(x_n, y_n) = 0$ and x_{n+1} from $g(x_{n+1}, y_n)$, which amounts to reversing the roles of f and g the ratio is inverted i.e.:

$$\left| \frac{\delta_{n+1}}{\delta_n} \right| = \left| \frac{f_x g_y}{f_y g_x} \right|$$

Thus convergence will always be obtained but rapid convergence only if one or other of the ratios is much less than unity.

This analysis was applied to both the methods described in detail above and, bearing in mind the order of magnitude of θ_1, θ_2 and θ_3 which can be seen fairly easily by inspection of Equations (12) and (13), it was found that method (1) gave rapid convergence when θ_3 was near zero or near 90° while method (2) was better when θ_3 was near 45°.

In method (1) for instance, putting $\theta_2^{(n)} = \theta_2 + \delta_n$ and $\theta_3^{(n)} = \theta_3 + \epsilon_n$

$$\left| \frac{\delta_{n+1}}{\delta_n} \right| \approx \frac{\sec^2 k\theta_3 \operatorname{cosec}^2 \theta_2}{\operatorname{cosec}^2 \theta_3 \operatorname{cosec}^2 k\theta_2}$$

$$\approx \frac{k^2 \sec^2 k\theta_3}{\operatorname{cosec}^2 \theta_3}$$

since θ_2 is small and hence

$$\operatorname{cosec}^2 \theta_2 \approx \frac{1}{\theta_2^2} \text{ and } \operatorname{cosec}^2 k\theta_2 \approx \frac{1}{k^2 \theta_2^2}$$

Thus
$$\left|\frac{\delta_{n+1}}{\delta_n}\right| \begin{array}{l}\text{is} \ll 1 \text{ when } \theta_3 \text{ is near zero}\\ \text{is} \gg 1 \text{ when } \theta_3 \text{ is near } 90^\circ\end{array}$$

Hence for θ_3 near 90°, the direction of the iteration cycle had to be reversed.

For method (2) Equation (C) is taken in the form:

$$\tan x_n\theta_1 = \cot y_n\theta_2 - \tan y_n\theta_3$$

$$y_{n+1} = x_n$$

Here
$$\left|\frac{\delta_{n+1}}{\delta_n}\right| = \frac{\theta_2 \operatorname{cosec}^2 y\theta_2 + \theta_3 \sec^2 y\theta_3}{\theta_1 \sec^2 x\theta_1}$$

with $y = x$.

Now, provided that θ_3 is not too near 90°, $\sec^2 y\theta_3 \ll \operatorname{cosec}^2 y\theta_2$ and $\sec^2 y\theta_1$, $\theta_1 \gg \theta_2$ and $\tan y\theta_1$ is fairly near $\cot y\theta_2$ by inspection of Equation (12) with y for k.

Thus in this case:
$$\left|\frac{\delta_{n+1}}{\delta_n}\right| \ll 1$$

3.5. Preparation of Graphs

Having found a large number of sets of k, θ_1, θ_2 and θ_3 and computed the corresponding N for each, graphs were drawn to facilitate reading the θ_1, θ_2 and θ_3 corresponding to any particular k and N. These are given in Figs. 4, 5 and 6. Only values of $k < 1$ were considered since, in practice, adding $\pi/2$ to the electrical length of the element θ_3 is equivalent to replacing k by $1/k$.

4. USE OF DESIGN CURVES

In order to illustrate the use of the filter design curves of Figs. 4, 5 and 6 three practical examples are considered.

Example 1

It is required to design a notch filter to suppress a frequency of 90 Mc/s and pass a frequency of 88 Mc/s. The pass loss is required to be less than 0·5 dB and the stop loss greater than 40 dB.

From Equations (7) (expressed graphically in Fig. 3) the resistances necessary at the pass and stop frequencies are:

$$\text{Pass frequency } Y_0R = 9{\cdot}0$$

$$\text{Stop frequency } Y_0r = 0{\cdot}005$$

From Equations (6a) and (6b) the design parameters of the filter must therefore be:

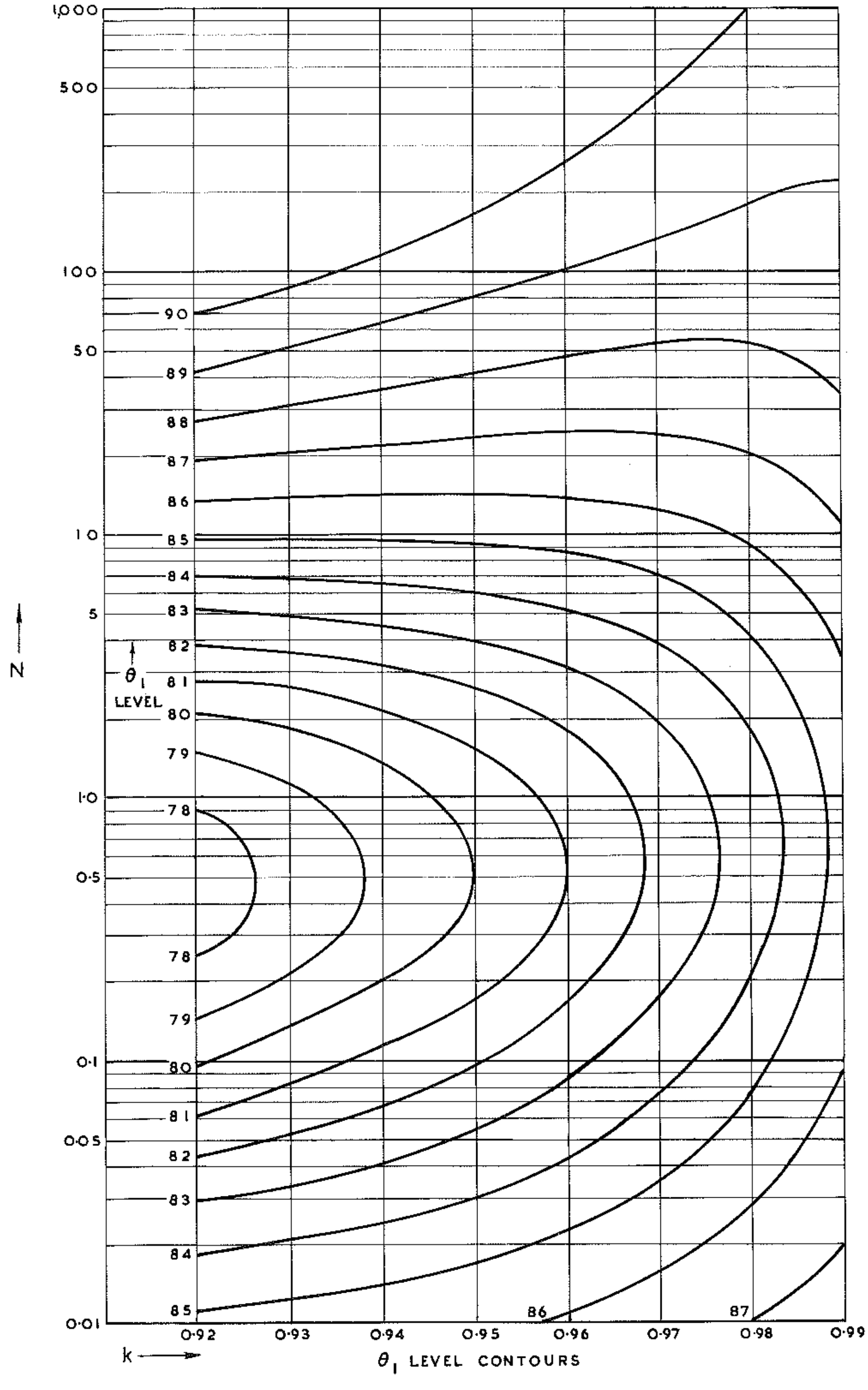
1,000
500
100
50
10
5
1·0
0·5
0·1
0·05
0·01
N
θ_1 LEVEL
90
89
88
87
86
85
84
83
82
81
80
79
78
78
79
80
81
82
83
84
85
86
87
0·92
0·93
0·94
0·95
0·96
0·97
0·98
0·99
k
θ_1 LEVEL CONTOURS

Fig. 4

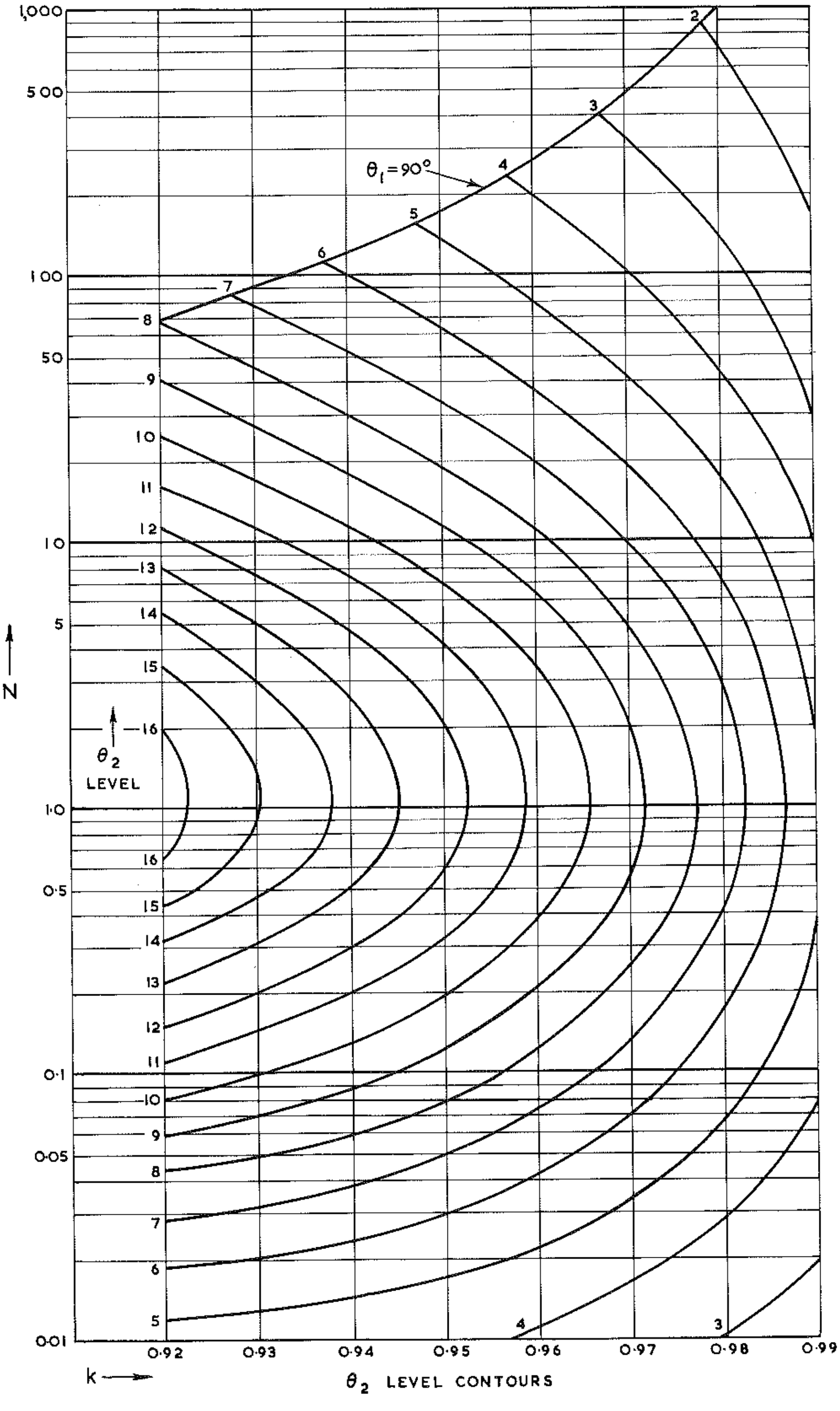
1,000
500
100
50
10
5
1·0
0·5
0·1
0·05
0·01
N
0·92
0·93
0·94
0·95
0·96
0·97
0·98
0·99
k
$\theta_1 = 90°$
θ_2 LEVEL
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
θ_2 LEVEL CONTOURS

Fig. 5

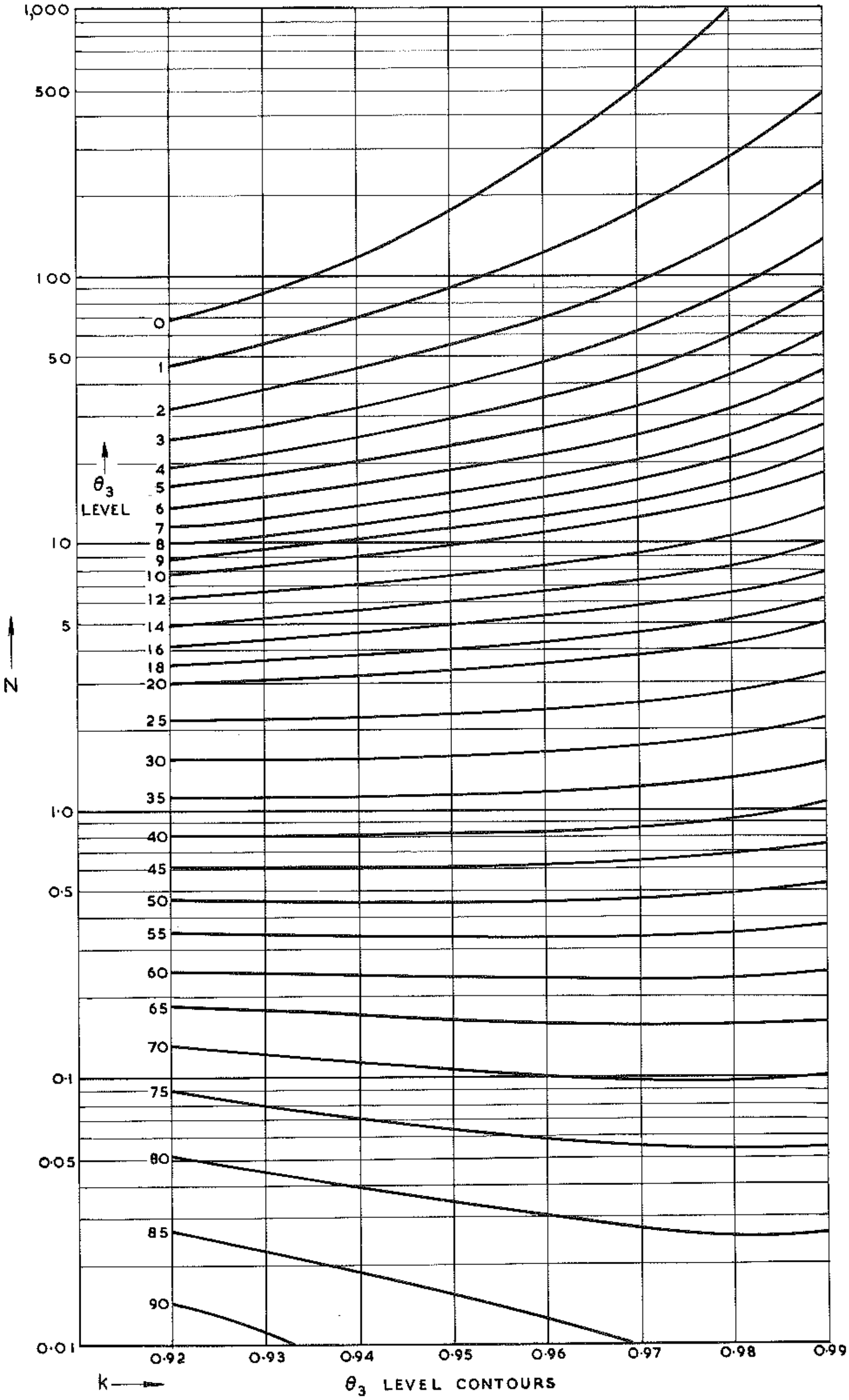
1,000
500
100
50
10
5
1·0
0·5
0·1
0·05
0·01
N
θ3 LEVEL
0
1
2
3
4
5
6
7
8
9
10
12
14
16
18
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
0·92
0·93
0·94
0·95
0·96
0·97
0·98
0·99
k
θ3 LEVEL CONTOURS

Fig. 6

$$N = \frac{1}{Y_o^2 Rr} = 22{\cdot}2$$

$$D = \frac{5460}{\lambda}\,(r/R)^{\frac{1}{2}}(1-k) = 0{\cdot}26\ \text{dB/100 ft}$$

1½ in. diameter copper feeder will have a loss approximately equal to that required and will therefore be satisfactory for construction of the filter. The filter may now be designed by use of the curves of Figs. 4, 5 and 6. The parameter k, the ratio of stop and pass frequencies, is 88/90 = 0•978 and using the values of $k = 0{\cdot}98$ and $N = 22{\cdot}2$, the lengths of the elements θ_1, θ_2 and θ_3 may be read off from the curves of Figs. 4, 5 and 6 respectively. The values measured at the stop frequency are:

$$\theta_1 = 87{\cdot}0^\circ$$
$$\theta_2 = 4{\cdot}8^\circ$$
$$\theta_3 = 7{\cdot}8^\circ$$

Example 2

If it is assumed that a filter of similar performance to that of Example 1 is required but with the pass and stop frequencies interchanged (i.e. pass frequency 90 Mc/s and stop frequency 88 Mc/s) the method of calculation must be modified. The modification is quite simple and can be described as follows: (a) Calculate the value of D as in Example 1. Since the required pass and stop losses are the same the value will be the same i.e. 0•29 dB/100 ft and 1½ in. diameter copper feeder will be suitable for construction of the filter. (b) Calculate the value of N as in Example 1 but when using the curves of Figs. 4, 5 and 6 use the inverse of the value calculated i.e.

$$\frac{1}{22{\cdot}2} = 0{\cdot}045$$

The values of the lengths of the elements now become:

$$\theta_1 = 85{\cdot}6^\circ$$
$$\theta_2 = 4{\cdot}6^\circ$$
$$\theta_3 = 77{\cdot}0^\circ$$

A filter with these dimensions will have a normalised conductance at the pass frequency equal to the required normalised resistance at the stop frequency and vice versa (by normalised is meant the values of rY_o and RY_o and their reciprocals).

It is necessary to add a quarter-wavelength to the θ_3 element in order to carry out the necessary inversion and to obtain the required resistances at the pass and stop frequencies. The values of the lengths of the elements in this case are thus:

$$\theta_1 = 85{\cdot}6^\circ$$
$$\theta_2 = 4{\cdot}6^\circ$$
$$\theta_3 = 167{\cdot}0^\circ$$

This solution is necessarily approximate, since the change of length with frequency of the additional quarter-wavelength of θ_3 has not been taken into account. The approximation is satisfactory in most practical cases, however, and it is intended, when time permits, to extend the design curves to cases where the pass frequency is higher than the stop frequency (i.e. θ_3 lies between 90° and 180°).

Example 3

The cases considered in Examples 1 and 2 are of filters in which the bandwidths of the signals being passed and stopped are small; for example VHF/FM applications, and the filter design is thus based purely on loss considerations. In passing it may be noted that the pass-loss specification may be based on the permissible dissipation in watts in the case of transmitting equipment and the noise factor of the receiver in the case of receiving equipment. In television applications, however, the bandwidth of the vision signal is of great importance and must be taken into account. In Fig. 7 a simple form of combining filter for television applications is shown and it is useful to consider the design of notch filter "A". This filter is required to pass a vision carrier signal with a loss not exceeding 3 dB at the edge of the vision band i.e. at vision carrier frequency minus 3 Mc/s and to stop the sound transmission at vision carrier frequency minus 3·5 Mc/s. If, in Equation (2), the losses are ignored (i.e. $\alpha = 0$ is assumed) then a value of N can be determined which satisfies the condition laid down for loss at the edge of the vision band. In Equation (4) we may eliminate r thus obtaining a value for R in terms of any chosen type of feeder used for the filter elements, i.e.

$$R = \frac{5460(1-k)}{\lambda Y_o D N^{\frac{1}{2}}}$$

The size of feeder required to give a specified pass loss at the sound carrier may thus be determined. The design of the filter from the curves of Figs. 4, 5 and 6 is then carried out as in the preceding examples.

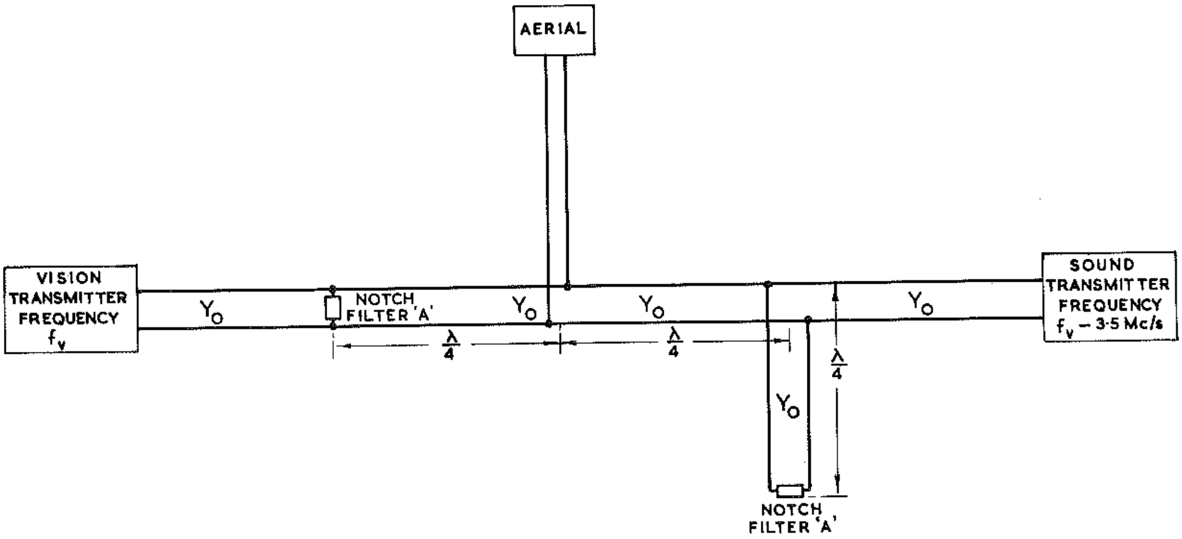

Fig. 7

5. CONCLUSIONS

The most convenient form of coaxial notch filter, both electrically and mechanically, is the "T"-type first described by B.M. Sosin.[1] The calculation of the dimensions of this type of filter for practical applications is difficult on account of the intractable nature of the equations governing its performance.

In this report a series of curves is presented together with basic design information which facilitates the rapid calculation of filters for specific applications.

6. REFERENCE

1. Sosin, B.M., "A Combining Filter for Vision and Sound Transmission", Proc.I.E.E., Vol. 99, Part IIIA, No. 18, pp. 253-264, April-May 1952.

MM

Printed by B.B.C. Research Department, Kingswood Warren, Tadworth, Surrey